Simulation Results

Guilherme Nettesheim

April 22, 2010

1 Introduction

I have attempted to create a simulation of Cherenkov radiation in the detector window that would give the parameters of photons incident to the photocathode, as well as the total number of photoelectrons released.

2 The Simulation

For a given wavelength, the simulation works by running through nx equally spaced points in the path of the charged particle. At each point it calculates the number, or probability, of a photon being emitted, and its trajectory. This is again repeated for $n\lambda$ equally spaced wavelengths in the effective range of the given photocathode, i.e, where the quantum efficiency is not zero.

2.1 Input and Output

The user gives the initial velocity of the charged particle in units of c and its charge in units of the elementary charge. The user also chooses from a list of given photocathodes and windows, and, lastly, chooses how fine to make the Riemann sum partitions.
The output is a log file of the following format

```
Simulation was run on 02.11.2010_16:25:02 by user USER
Initial velocity: (0.000000, 0.000000, 1.000000), Charge: 1e
Window material: Fused Silica 2.0mm, Window thickness: 2.000000 mm, Index of Refraction (160 nm): 1.640296
Photocathode: QE58
Number of divisions of charged particle path: 50, Number of divisions of wavelength of Cherenkov light: 50
22.221469 photoelectrons created

744.308079 750.101925 7.545222 0.000000 0.000000 0.000000 1.084721 0.000000 160.000000 2262.000755 0.000000 0.000000
308.496274 956.749838 7.481033 0.000000 40.000000 0.133426 1.096893 0.000000 160.000000 2202.757409 40.000000 0.000000
-442.680598 585.436157 7.123279 0.000000 80.000000 0.266851 1.205665 0.000000 160.000000 2055.505195 80.000000 0.000000
374.842378 922.073574 7.495967 0.000000 120.000000 0.400277 1.083873 0.000000 160.000000 2127.234468 120.000000 0.000000
...
```

Where the columns correspond to photon incidence x-coordinate, photon incidence y-coordinate, photon incidence time, photon emission x-coordinate, photon emission z-coordinate, photon emission time, omega angle, zeta angle, wavelength, photon distance, particle distance, and number of electrons released. Each row corresponds to a certain wavelengths, and a certain distance in the charged particle's path.

2.2 Calculating the number of photoelectrons released

2.2.1 Cherenkov Radiation formula

We begin with the following formula for Cherenkov radiation[1] per path length per angular frequency, where I is the total energy, x is the path length of the charged particle, ω the angular frequency of the photons emitted, Q the charge of the particle, $\epsilon(\omega)$ the dielectric constant, and *beta* the velocity of the charged particle.

$$\frac{\partial^2 I}{\partial x\,\partial\omega} = \frac{Q^2\omega}{c^2}\left[1 - \frac{1}{\beta^2\epsilon(\omega)}\right]$$

The dielectric constant is roughly constant for all wavelengths which apply, and we use instead the index of refraction $n = \sqrt{\epsilon}$. We also would like to calculate the number of photons, so we divide by $E = \hbar\omega$:

$$\frac{\partial^2 I}{\partial x\,\partial\omega} = \frac{Q^2}{\hbar c^2}\left[1 - \frac{1}{\beta^2\epsilon}\right] = \frac{\alpha q^2}{c}\left[1 - \frac{1}{\beta^2 n^2}\right]$$

Where $Q = qe$, and α is the fine-structure constant.
Now making the substitutions $\omega = \frac{2\pi v}{\lambda} = \frac{2\pi c}{\lambda n}$ and $d\omega = \frac{-2\pi c}{n}\frac{d\lambda}{\lambda^2}$, we have the following formula in terms of the wavelength of Cherenkov radiation:

$$dN_{photons} = \left(\frac{2\pi\alpha q^2}{n\lambda^2}\right)\left[1 - \frac{1}{\beta^2 n^2}\right]dx\,d\lambda$$

We switch the order of summation so that our formula is positive.

2.2.2 Quantum Efficiency and Transmission

We must also factor in the quantum efficiency of the photocathode, and the transmission of the window. The photocathodes used have a narrow band of effectiveness, between 100 and 900 nanometers, where the quantum efficiency[2] is not zero. We thus only consider wavelengths within this bound, as light of any other frequency has no effect on the photocathodes. Likewise, only a certain percentage of the light emitted by the charged particle reaches the photocathode, per distance traveled. The transmission percentage usually approaches 100% for wavelengths longer than 300 nanometers, while decreasing to zero for wavelengths shorter than 100 nanometers.
The simulation creates functions $Trans(x_p, \lambda)$ and $QE(\lambda)$. From manufacturer data for given a photocathode and window material, these functions extrapolate the the quantum efficiency and transmission for a given photon's path length from emission to incidence with the photocathode, x_p, and its wavelength, λ.
Thus we can express the total number of photoelectrons created as:

$$N_{photons} = \int dN_{electrons}\,Trans(x_p, \lambda)\,QE(\lambda)$$

[1] J.D. Jackson, *Classical Electrodynamics*, Second Edition, Equation 14.133
[2] $QE = \frac{Electronsreleased}{Incidentphotons}$

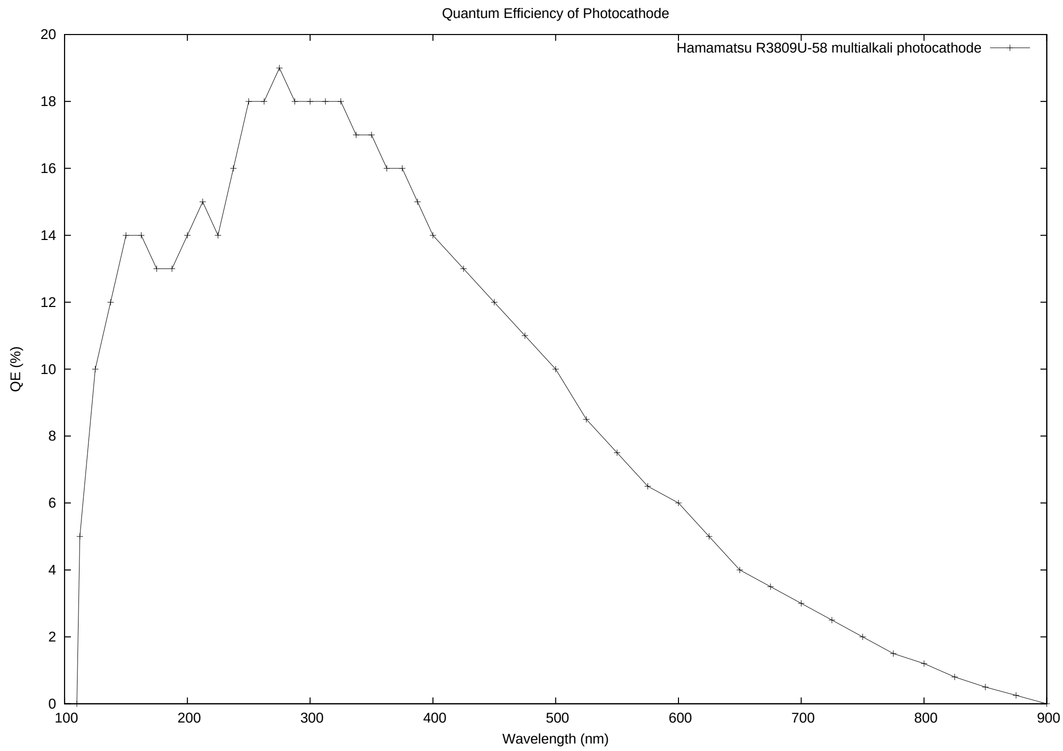


Figure 1: Quantum efficiency of the Hamamatsu R3809U-58 multialkali photocathode used in this simulation

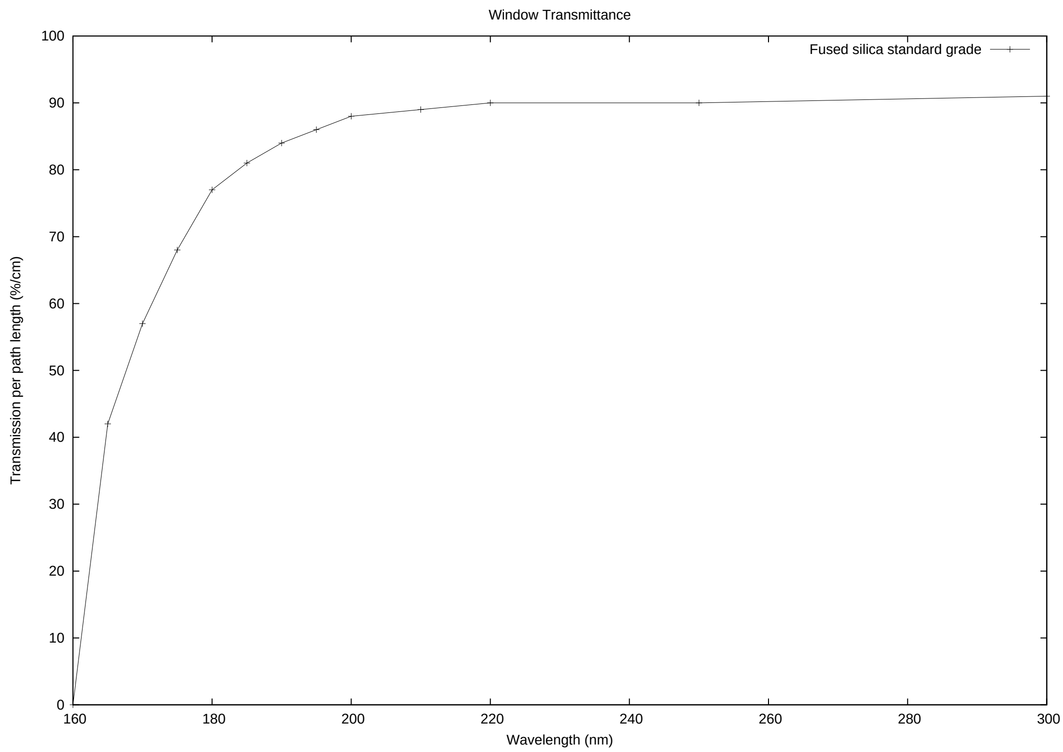


Figure 2: Transmittance of fused silica standard grade

2.2.3 Integration

The total number of electrons is therefore given by

$$N = \int\int \left(\frac{2\pi\alpha q^2}{n\lambda^2}\right)\left[1 - \frac{1}{\beta^2 n^2}\right] Trans(x_p, \lambda)\, QE(\lambda)\, d\lambda\, dx$$

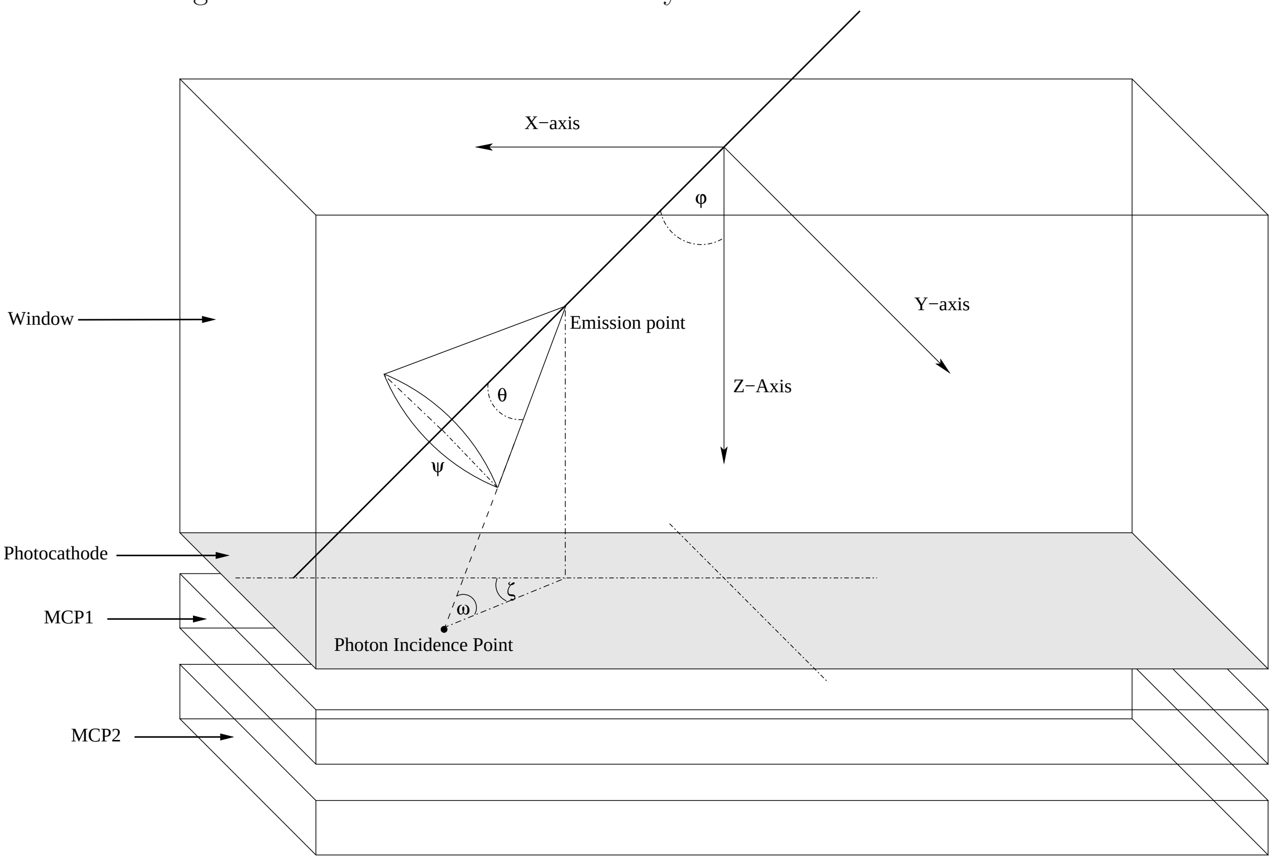
We approximate this integral by a double left Riemann sum. The simulation partitions the spectrum of wavelengths in which the Quantum Efficiency is nonzero into $n\lambda$ frequencies, and the total distance traveled by the charged particle into nx distances. This gives us an approximation for the total number of photoelectrons created.

2.3 Geometry and photon/electron properties

For each data point, we calculate the time and position of the photon's emission along the charged particle's path, the time and position of its incidence with the photocathode, and the two angles which characterize the photon's incidence with the photocathode. The time components fall naturally from the spatial ones, as we know the charged particle's initial velocity, and the photon's in the glass.

2.3.1 Coordinate System

We define a right-handed Cartesian coordinate system as follows:



We place the origin at the point where the particle enters the detector window, and rotate the coordinate system so that the particle moves in the x-z plane. This coordinate system can easily be extended to the entire detector, but for our purposes we only apply it to the first layer, the window and the photocathode. Therefore, using a 2.5mm thick window, the photocathode lies in the plane $(x, y, 2.5mm)$.
We define also five angles:

- ϕ is the angle the particle trajectory makes with the z-axis, and is determined by the particle's initial velocity.
- θ is the Cherenkov angle, the half-angle of the cone of Cherenkov radiation emitted along the particle's path ,which we calculate with the formula $\theta = \arccos \frac{1}{\beta n}$
- For the path of a given photon emitted from this cone of Cherenkov radiation, the angle around is cone is denoted ψ, with $\psi = 0$ when the photon's path lies in the x-z plane. This angle is randomly generated for each photon, for an even distribution.
- ω is the angle between the photocathode plane and the given photon's trajectory.
- ζ is the angle between the photon path's projection onto the photocathode plane and the x-axis.

Together, these angles completely characterize the path of the charged particle and photons emitted along it.

2.3.2 Calculations

Given a point along the charged particle's track, the Cherenkov angle θ, and the randomly generated angle ψ, we can calculate the trajectory of the emitted photon, and its point of intersection with the photocathode plane. This geometrical problem was solved in the previous simulation attempt[3], and this is the calculation which this simulation uses.
In addition, we calculate the angles ζ and ω mentioned above. ζ is the angle between the projection of the photon path along the photocathode plane and the x-axis, and the ω angle is the angle between this same photon path and the photocathode plane.

3 Results

We run the simulation with the following parameters:

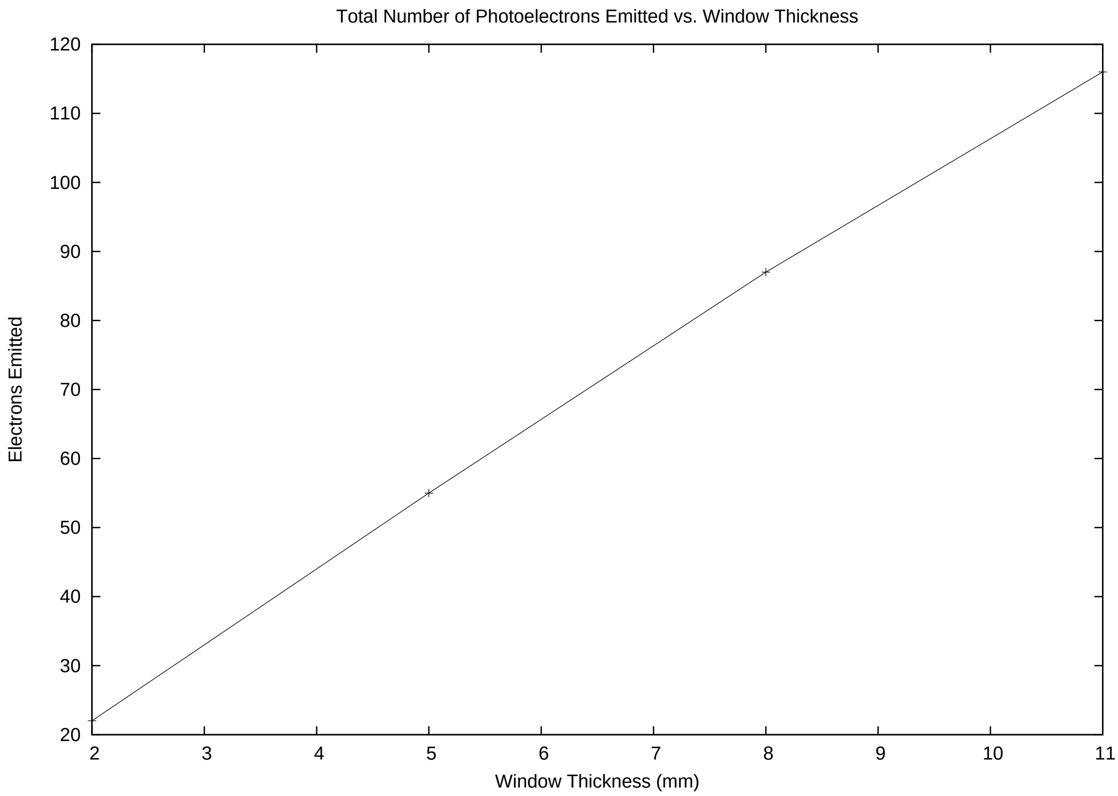
- The initial velocity is (0,0,1), that is, the charged particle moves at c normal to the window surface
- The charge is 1, that is, it has elementary charge
- We use fused silica standard grade[4]
- We use a Hamamatsu R3809U-58 multialkali photocathode[5]

[3] S. Bhat, T. Credo, et al., *Simulation of a Picosecond TOF Device*, p. 20
[4] See http://psec.uchicago.edu/glass/HPFSFusedSilicaStandardGradeCorning.pdf
[5] See http://usa.hamamatsu.com/assets/pdf/catsandguides/PMTCAT_special_mcp-pmt.pdf

- We use partitions of 100 for both the path length and the wavelength spectrum. Running the simulation with higher numbers shows that this is a fine enough partition to give a good approximation.

3.1 Total number of photoelectrons

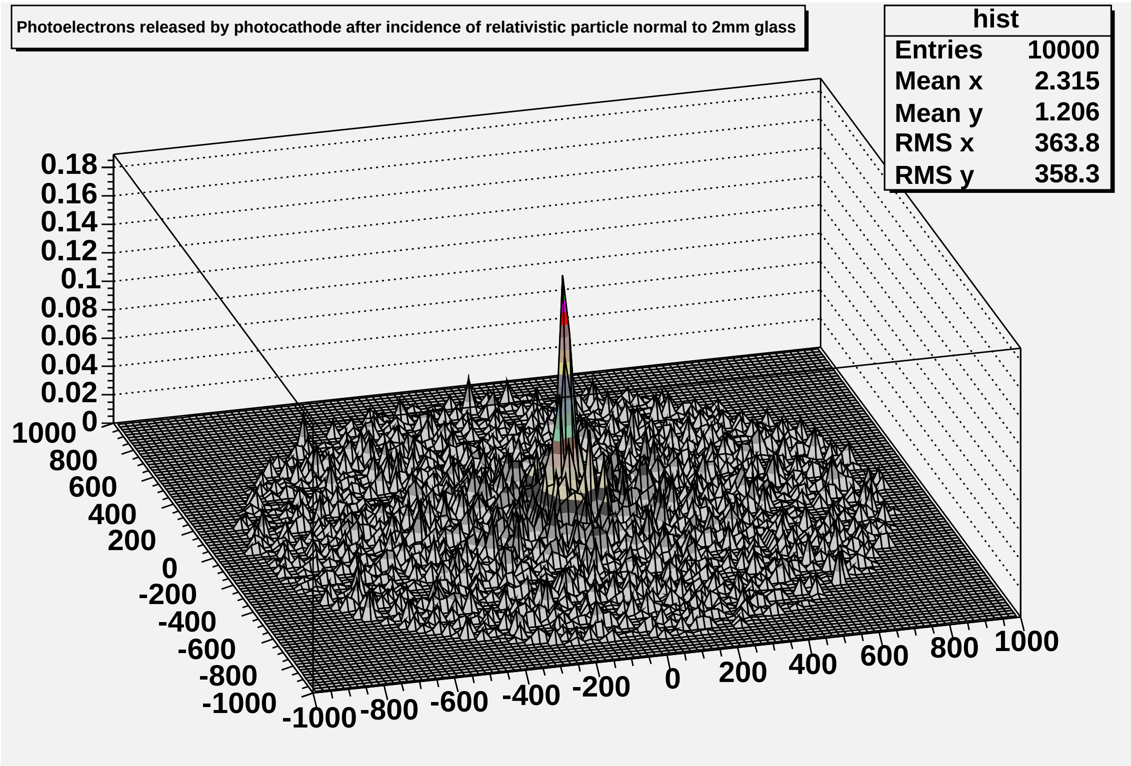
Window thickness (mm)	Photoelectrons ± 1
2.0	22
5.0	55
8.0	87
11.0	116

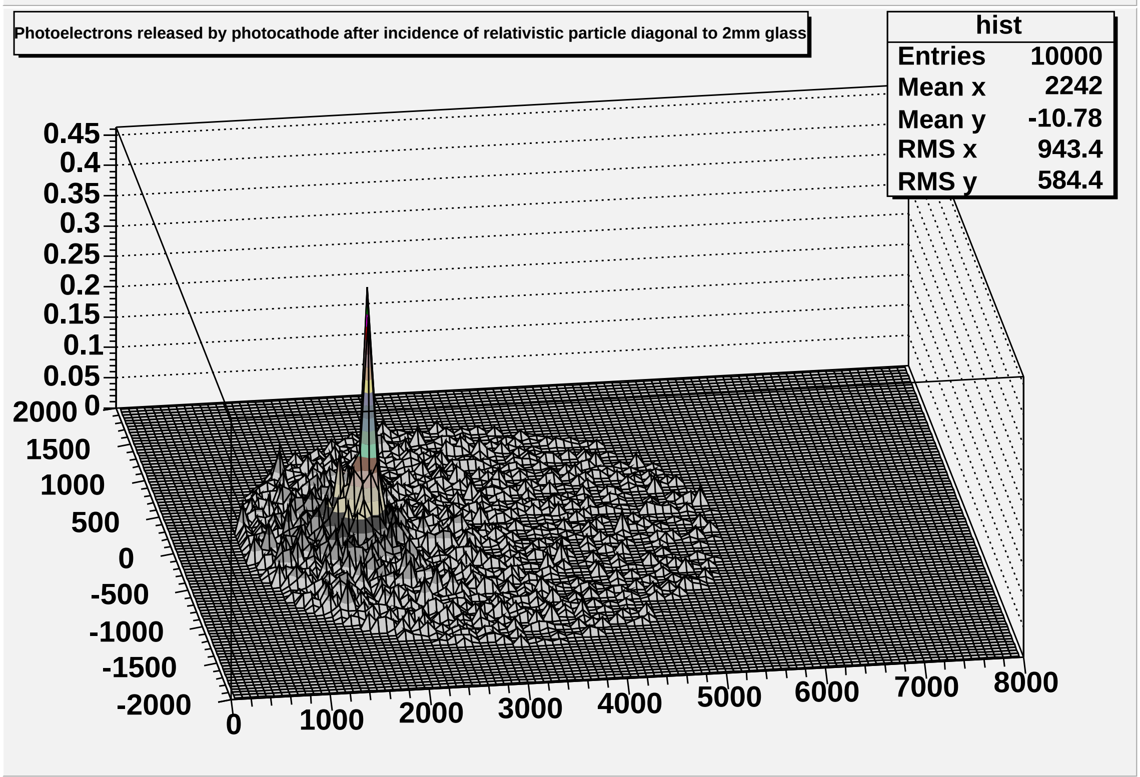


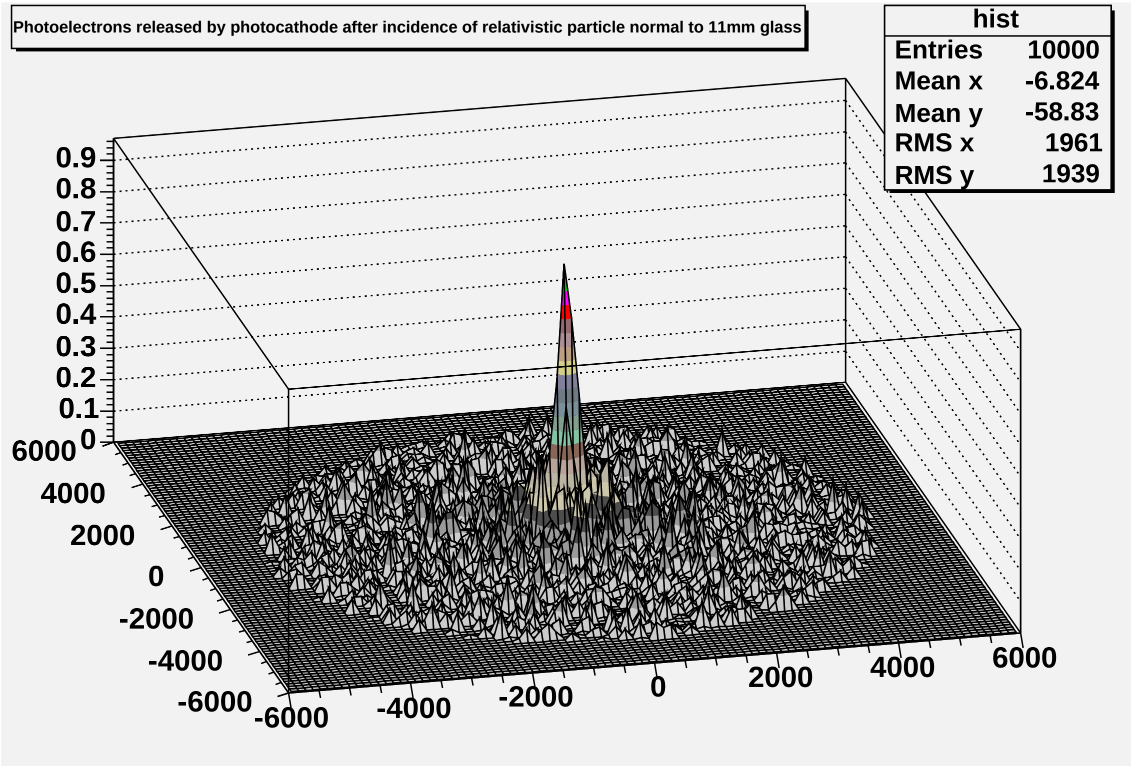
These results agree with previous simulation efforts by Tim Credo and we see that the total number of photoelectrons released varies roughly linearly wth window thickness.

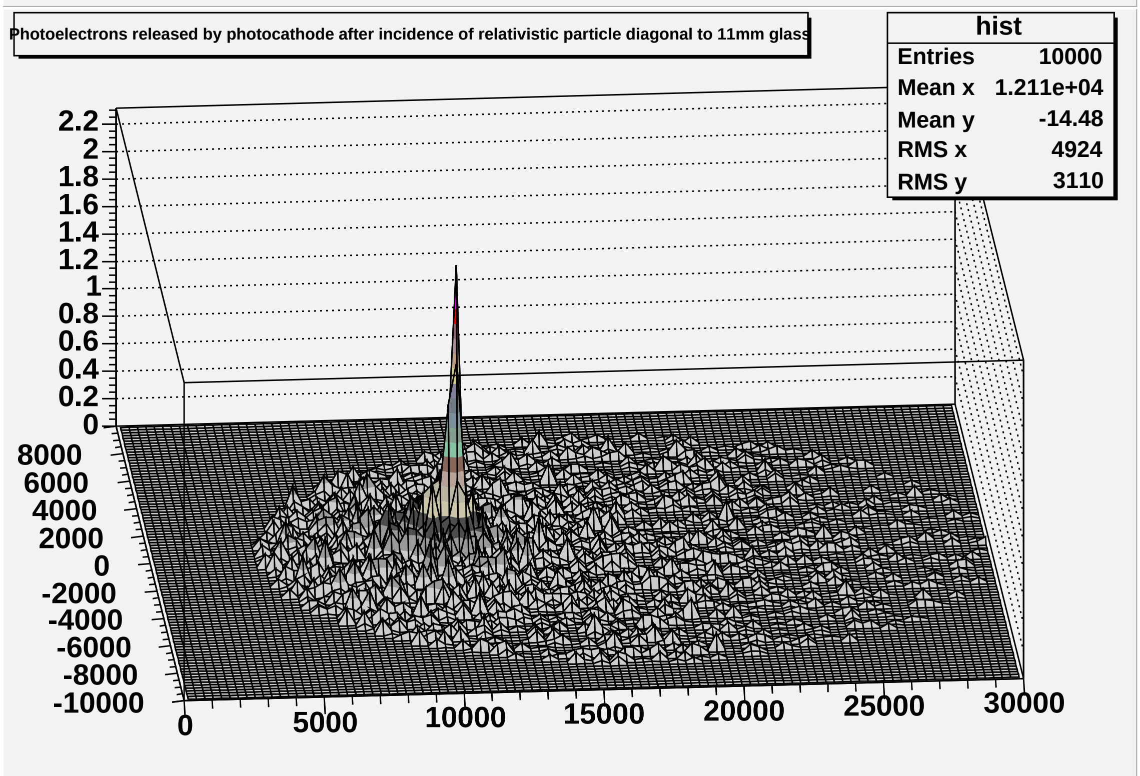
3.2 Position

Below we present plots of the simulation results. We again use micrometers for spatial measurements. By relativistic we mean a speed of c, whether the particle is incident normal to the glass or diagonal, that is, at a 45°.

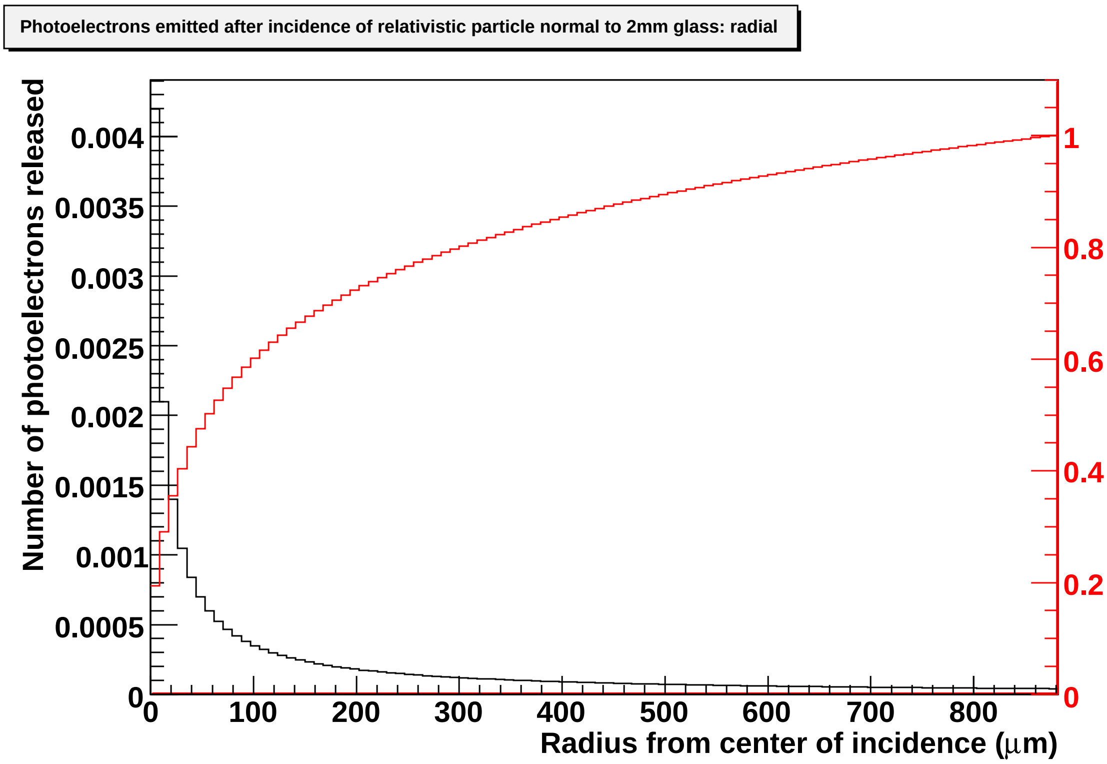

Photoelectrons released by photocathode after incidence of relativistic particle normal to 2mm glass
hist
Entries 10000
Mean x 2.315
Mean y 1.206
RMS x 363.8
RMS y 358.3
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1000
800
600
400
200
0
-200
-400
-600
-800
-1000
-1000 -800 -600 -400 -200 0 200 400 600 800 1000

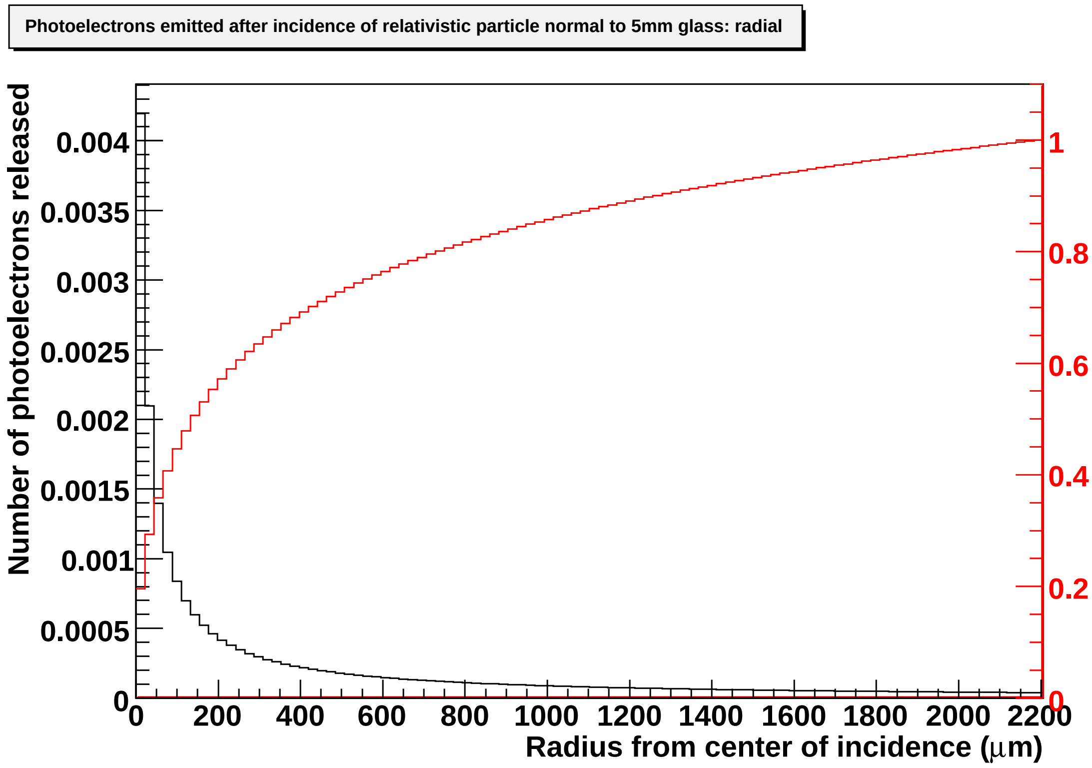

Photoelectrons released by photocathode after incidence of relativistic particle diagonal to 2mm glass
hist
Entries 10000
Mean x 2242
Mean y -10.78
RMS x 943.4
RMS y 584.4
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
2000
1500
1000
500
0
-500
-1000
-1500
-2000
0 1000 2000 3000 4000 5000 6000 7000 8000

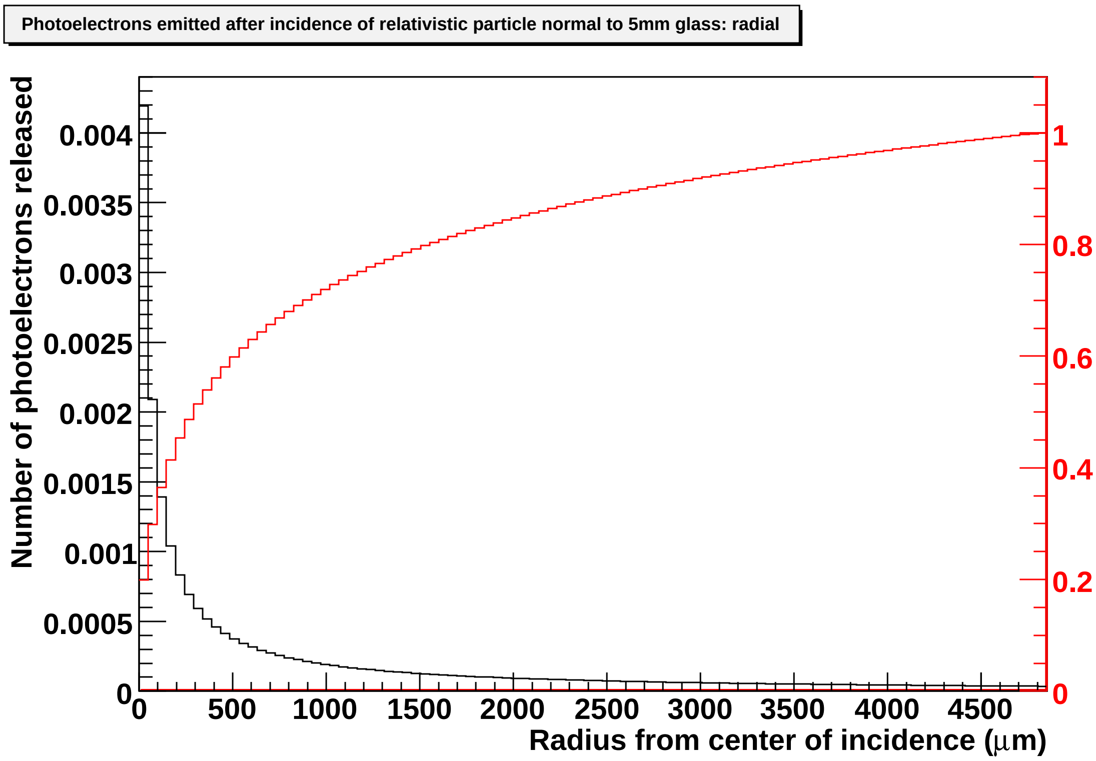




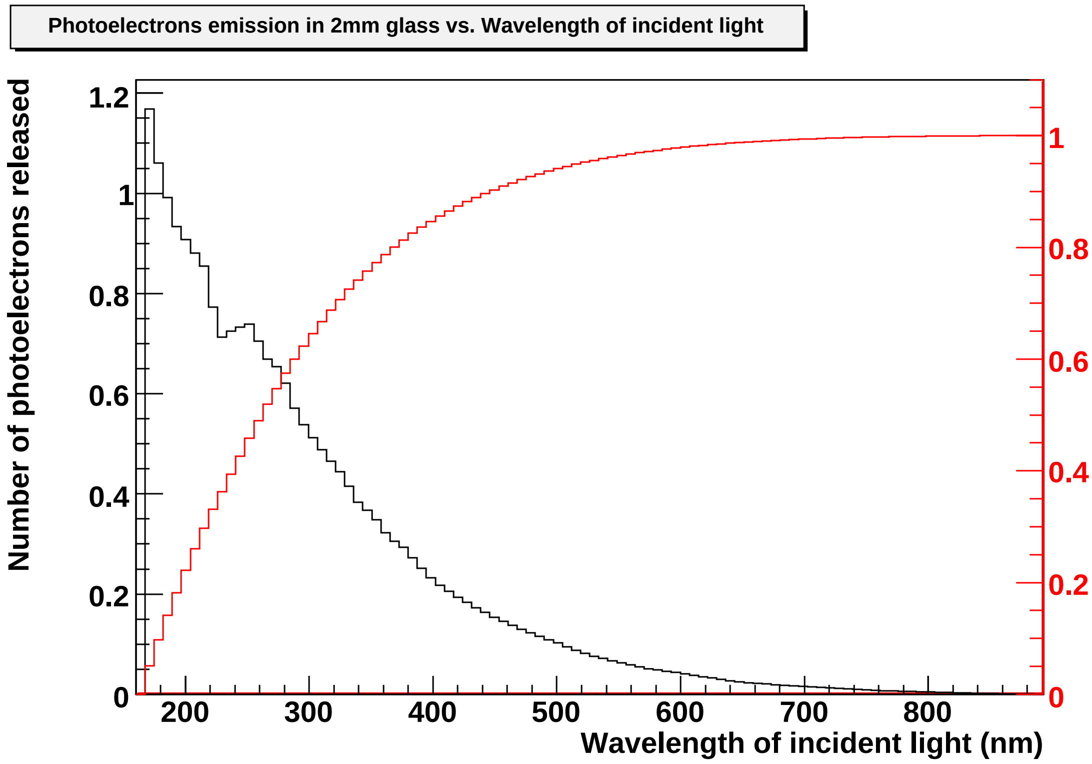
We also present a one-dimensional plot of the number of photoelectrons emitted as a function of the radius as measure from the center of the shower, in this case $(0, 0, z)$.

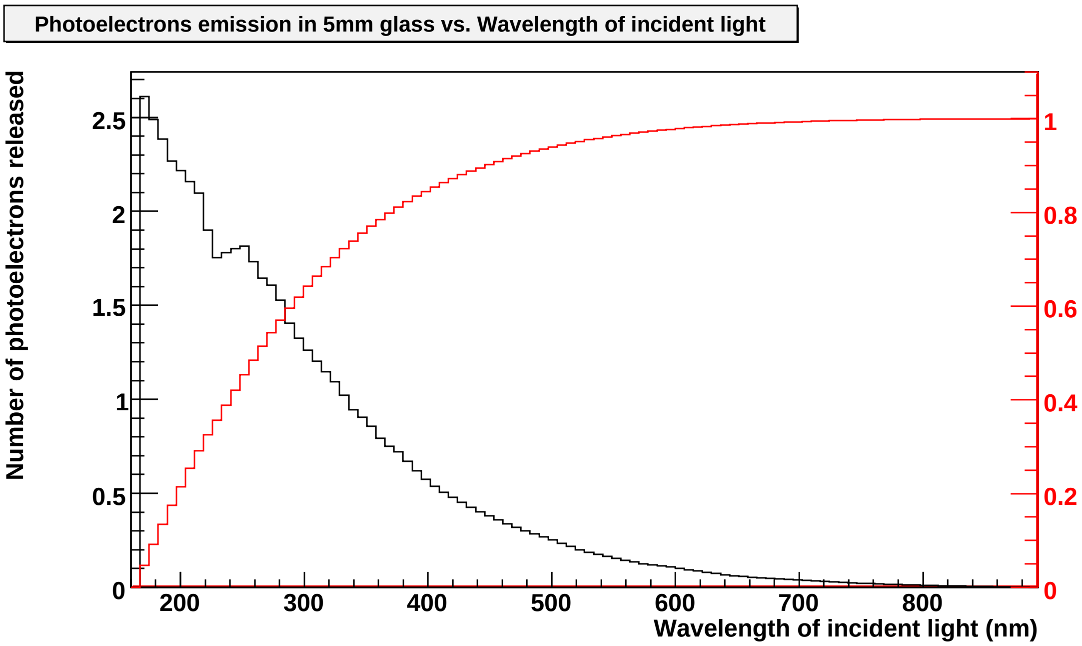

Photoelectrons emitted after incidence of relativistic particle normal to 2mm glass: radial
Number of photoelectrons released
0.004
0.0035
0.003
0.0025
0.002
0.0015
0.001
0.0005
0
0
100
200
300
400
500
600
700
800
Radius from center of incidence (μm)
1
0.8
0.6
0.4
0.2
0

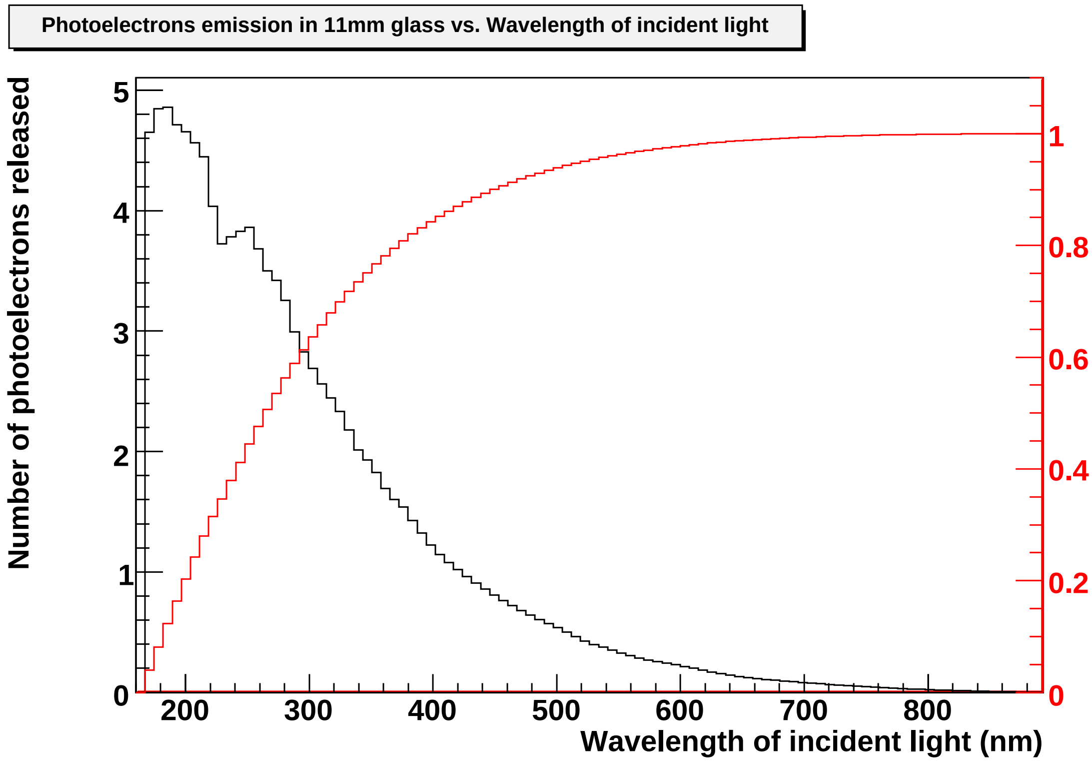

Photoelectrons emitted after incidence of relativistic particle normal to 5mm glass: radial
Number of photoelectrons released
0.004
0.0035
0.003
0.0025
0.002
0.0015
0.001
0.0005
0
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
Radius from center of incidence (μm)
1
0.8
0.6
0.4
0.2
0



And finally we plot the number of photoelectrons emitted as a function of the wavelength of incident light.




Photoelectrons emission in 5mm glass vs. Wavelength of incident light
Number of photoelectrons released
Wavelength of incident light (nm)
0
0.5
1
1.5
2
2.5
200
300
400
500
600
700
800
0.2
0.4
0.6
0.8


Photoelectrons emission in 11mm glass vs. Wavelength of incident light
Number of photoelectrons released
Wavelength of incident light (nm)
0
1
2
3
4
5
200
300
400
500
600
700
800
0.2
0.4
0.6
0.8