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Simulation of the Fermilab Test Beam

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1 Introduction

The Fermilab Test Beam Facility (FTBF) at Fermi National Accelerator Laboratory (Fermilab) is a flexible test-beam facility with a controlled particle beam. The test beam consists of a series of dipole magnets, quadrupole magnets, and collimators. An LAPPD-based time-of-flight (ToF) system will be implemented at FTBF in order to provide users a method of particle identification. This involves stationing two LAPPDs in the beamline and measuring the time it takes for particles to pass from one detector to another. By measuring this time as well as the path length the particle takes, we can infer the velocity of the particle. Since we have the momentum of the beam, this allows us to measure the mass of the particle, and thus identify what type of particle it is. [1]

The motivation for this study was primarily to understand the effect of dispersion in the beam; that is, how the x positions of particles are dependent on deviations in momentum, as this will ultimately affect time of flight.

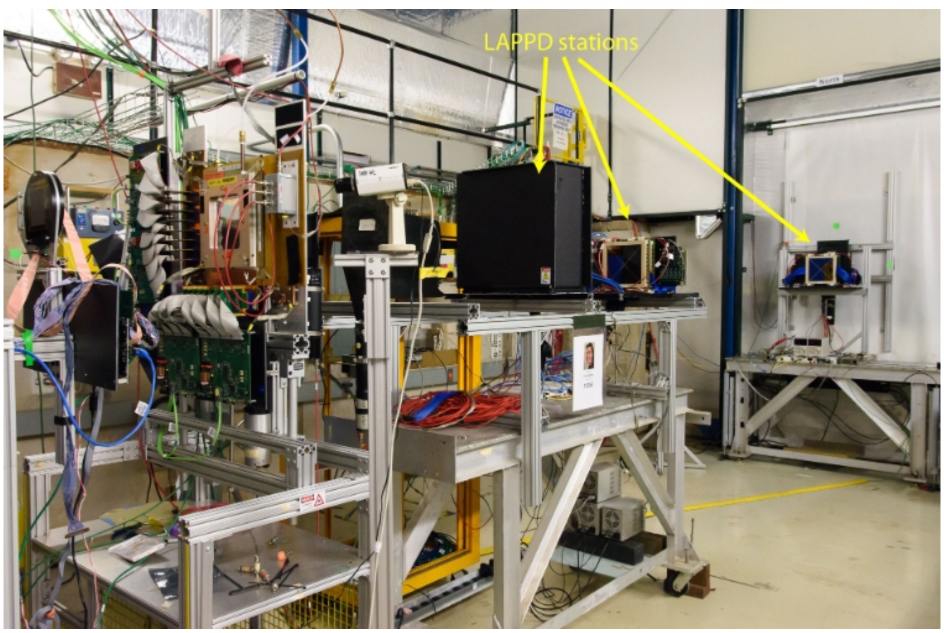


Figure 1: Photo of the LAPPD ToF system in MTest[1]

2 About the Test Beam

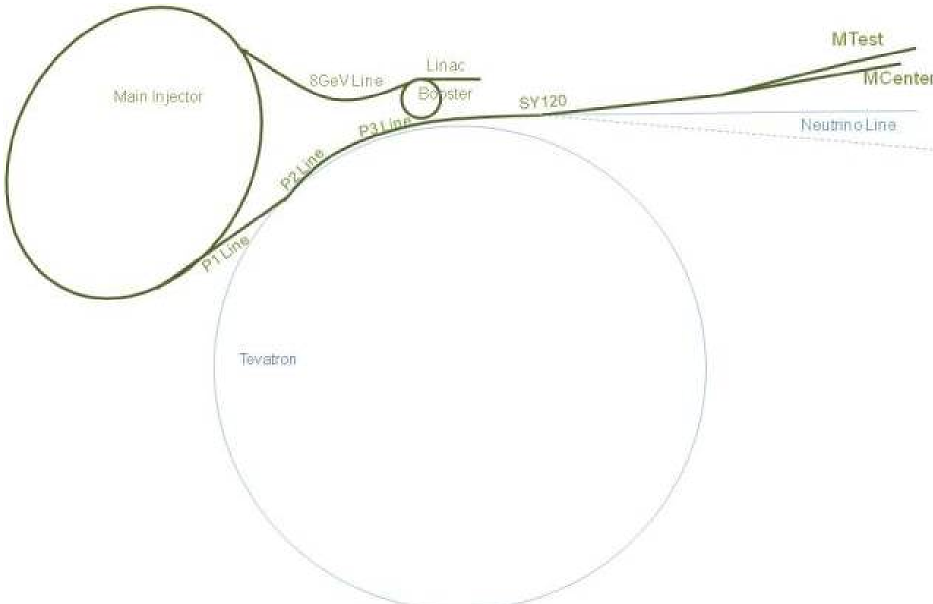


Figure 2: Beam delivery path (Source: FTBF)

The beam is created by extracting particles from the Main Injector, which is the circular track of the beam that accelerates particles to a specific energy and frequency. From there, beam is extracted in cycles and sent along offshoots, including to the test beam (labeled MTest).

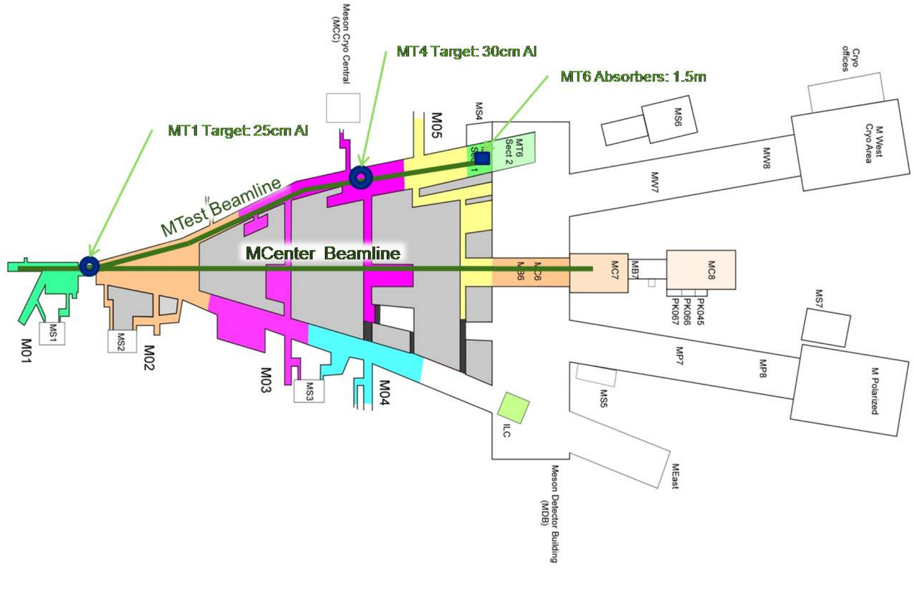


Figure 3: Diagram of MTest (Source: FTBF)

MTest begins at the MT1 target and is divided into six sections, ending at the MT6 absorber. The LAPPDs will be stationed in the far downstream area of MTest, in the MT5 and MT6 regions. This is the area of MTest most commonly used for experiments. [5]

This simulation begins at the MT4 target, which is an aluminum target used to produce secondary particles. There are several beam modes available, including high energy pions, low energy pions, and muons. For this study, we focused on the low energy pion mode, which produces positive pions in the 1-32 GeV range.

2.1 Diagram

The following diagram shows the active elements of this section of the beamline.

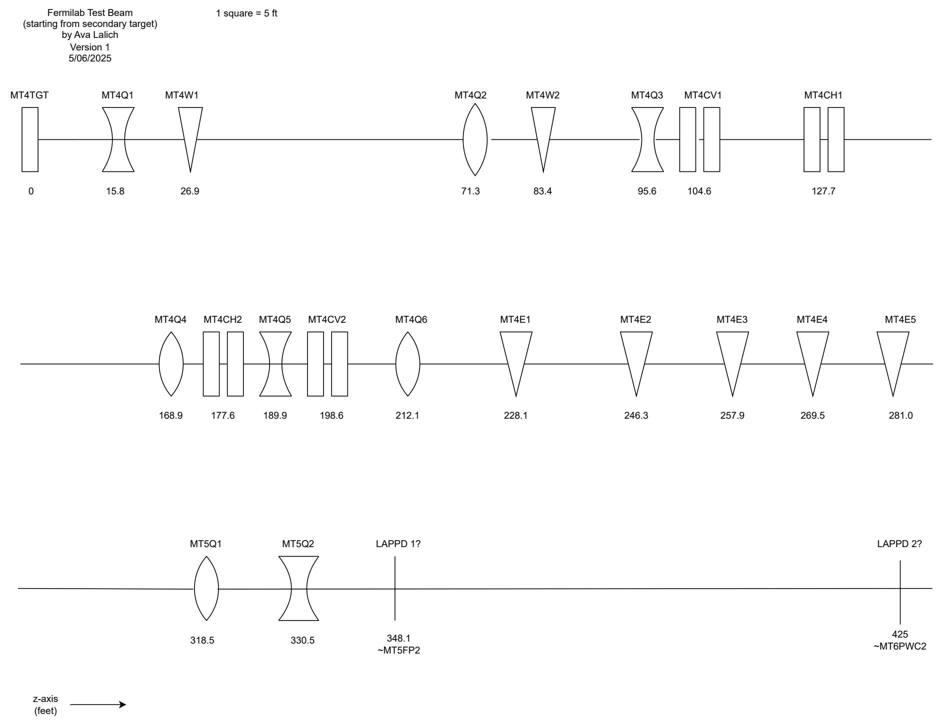


Figure 4: A simple diagram of MTest starting from the MT4 target, including its quadrupoles, dipoles, and collimators, as well as the approximate locations of where LAPPDs will be placed.

2.2 Coordinates

These coordinates refer to each element's location down the z-axis of the beamline, taken from the beam sheet[4]. The coordinates from this file are with respect to some arbitrary reference point, so the rightmost column expresses the coordinates with the MT4 target as the starting point.

No.	Name	Element Type	Coordinate (m)	Coordinate w.r.t target (m)
0	MT4TGT	Target	36.031	0
1	MT4Q1	Quad	40.850	2.269
2	MT4W1	Dipole	44.230	5.653
3	MT4Q2	Quad	57.771	19.202
4	MT4W2	Dipole	61.461	22.885
5	MT4Q3	Quad	65.165	26.594
6	MT4CV1	Collimator	67.927	32.239
7	MT4CH1	Collimator	74.963	36.640
8	MT4Q4	Quad	87.520	46.472
9	MT4CH2	Collimator	90.165	49.895
10	MT4Q5	Quad	93.921	51.163
11	MT4CV2	Collimator	96.565	54.552
12	MT4Q6	Quad	100.686	56.199
13	MT5E1	Dipole	105.570	63.087
14	MT5E2	Dipole	111.101	66.626
15	MT5E3	Dipole	114.632	70.141
16	MT5E4	Dipole	118.163	73.685
17	MT5E5	Dipole	121.694	77.216
18	MT5Q1	Quad	133.119	98.651
19	MT5Q2	Quad	136.776	102.296
20	LAPPD 1	Detector	145.188	110.708
21	LAPPD 2	Detector	168.627	134.147

Table 1: Element coordinates

More information on these elements can be found in Section 3.1 as well as the **beam sheet**[8]. Additionally, a 2015 survey of MTest including photos and geometry of the above elements can be found **here**[7].

3 Overview of Transport

Transport[2] is a beamline simulation framework that models optical elements using matrices. The program represents particles using the following six-vector:

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ l \\ \delta \end{pmatrix}$$

where x and y represent the horizontal and vertical displacements, x' and y' represent the particle's angle with respect to the beamline, l represents the location down the beamline, and $\delta = (p - p_0)/p_0$ (dispersion). Additionally, each element in the beamline is represented by a 6x6 matrix, which represents how the particle's six-vector is transformed as it passes through that element. We can multiply several of these matrices together to get one matrix that describes how a particle will transform after passing through all of those optical elements. Another point to note is that the transfer matrix allows us to clearly see a coupling between two components- for example, the (1,6) entry of a transfer matrix shows us the dependence of x on dp/p.

The user creates the beamline by writing an input file, in which each element is specified by a "type code" and several parameters. From this input file, the transfer matrices are calculated. In this study, only first-order effects were modeled, although Transport allows for higher-order effects.

3.1 Simulation Parameters

For this study, we used values corresponding to a $\pi+$ beam at 1GeV with a momentum byte of 0.5%. The magnetic fields of quadrupoles and dipoles scale linearly with beam momentum, so the following values are with respect to this reference momentum. These values were taken from the FTBF's G4Beamline simulation of this section of MTest[4]. Physically, we set the initial beam to have a spatial width of 1 mm and an angular divergence of 1mrad. Below are the necessary parameters to describe each active element of the beamline (not including collimators).

Drifts

Drifts are portions of space in between elements in the beam lattice where particles drift freely, subject to no external forces. In Transport, these are specified by a type code 3. The only parameter required is the length of the drift.

$$\begin{pmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

L is the length of the drift.

Drift Input Parameters

1. Length (m)

No.	Length (m)
0	2.269
1	0.336
2	10.501
3	0.635
4	0.661
5	2.597
6	4.401
7	9.832
8	0.375
9	1.268
10	0.341
11	1.647
12	3.84
13	0.491
14	0.467
15	0.496
16	0.483
17	18.387
18	0.597

Table 2: Drift lengths

Dipoles (Bends)

Dipoles are magnets with two poles used to bend the trajectory of a particle. They are represented in Transport using a type code 4 and require the following parameters: effective length, magnetic field strength, and the gradient of the magnetic field.

$$\begin{pmatrix} \cos(kL) & \frac{1}{k_x}\sin(k_xL) & 0 & 0 & \frac{h}{k_x}\sin(k_xL) & \frac{h}{k_x^2}(1-\cos(k_xL)) \\ -k_x\sin(k_xL) & \cos(k_xL) & 0 & 0 & h(1-\cos(k_xL)) & \frac{h}{k_x}\sin(k_xL) \\ 0 & 0 & \cos(k_yL) & \frac{1}{k_y}\sin(k_yL) & 0 & 0 \\ 0 & 0 & -k_y\sin(k_yL) & \cos(k_yL) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where L is the effective length of the dipole in meters and k_x,k_y are the magnet's focusing strength in the x and y directions.

Dipole Input Parameters

1. Effective Length (m): 3.048 for all bends
2. B-field along trajectory (kG)

Name	Value
MT4W1	0.083125
MT4W2	0.090625
MT5E1-5	-0.098281

Table 3: Dipole B fields

3. Field gradient: 0 for all bends

Quadrupoles

Quadrupoles are magnets that create a field with four poles. Quadrupoles are either horizontally or vertically focusing, meaning that they focus the beam in one direction and defocus in the other. However, when alternating horizontally and vertically focusing quadrupoles are placed in a series, with drifts between them, it has a net focusing effect in both directions. These are specified in Transport using a type code 5. They require the following parameters: effective length (meaning the length of the field, not the physical length), magnetic field strength at pole tip, and aperture radius.

$$\begin{pmatrix} \cos(k_q L) & \frac{1}{k_q}\sin(k_q L) & 0 & 0 & 0 & 0 \\ -k_q \sin(k_q L) & \cos(k_q L) & 0 & 0 & 0 & 0 \\ 0 & 0 & \cosh(k_q L) & \frac{1}{k_q}\sinh(k_q L) & 0 & 0 \\ 0 & 0 & k_q \sinh(k_q L) & \cosh(k_q L) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where L is the effective length of quadrupole (meters) and k_q is the focusing strength. The (0,1) and (2,3) entries will be flipped if the quadrupole is vertically focusing.

Quad Input Parameters

1. Effective Length (m): 3.048 for all quads
2. B field strength at pole tip (kG):

Name	Value
MT4Q1	1.9415
MT4Q2	-1.131625
MT4Q3	-1.131625
MT4Q4	0.38075
MT4Q5	-0.56825
MT4Q6	0.607075
MT5Q1	0.159593
MT5Q2	-0.319185

Table 4: Quadrupole B fields

3. Aperture Radius (cm): 3.65 for all quads

4 Outputs

The output of Transport is a text file that contains the total transfer matrix of the beam at the location of each element. The two matrices relevant to this study are the matrices at the locations of LAPPD 1 and LAPPD 2, which are 110.708 and 134.174 m down the beamline, respectively.

At LAPPD 1:

$$\begin{pmatrix} 0.87567 & 11.73389 & 0.00000 & 0.00000 & 0.00000 & -0.00022 \\ -0.00294 & 1.10259 & 0.00000 & 0.00000 & 0.00000 & -0.00029 \\ 0.00000 & 0.00000 & 1.04924 & 10.35945 & 0.00000 & 0.00000 \\ 0.00000 & 0.00000 & -0.00598 & 0.89407 & 0.00000 & 0.00000 \\ 0.00003 & 0.00032 & 0.00000 & 0.00000 & 1.00000 & 0.00000 \\ 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 1.00000 \end{pmatrix}$$

At LAPPD 2:

$$\begin{pmatrix} 0.86879 & 14.31825 & 0.00000 & 0.00000 & 0.00000 & -0.00090 \\ -0.00294 & 1.10259 & 0.00000 & 0.00000 & 0.00000 & -0.00029 \\ 0.00000 & 0.00000 & 1.03523 & 12.45507 & 0.00000 & 0.00000 \\ 0.00000 & 0.00000 & -0.00598 & 0.89407 & 0.00000 & 0.00000 \\ 0.00003 & 0.00032 & 0.00000 & 0.00000 & 1.00000 & 0.00000 \\ 0.00000 & 0.00000 & 0.00000 & 0.00000 & 0.00000 & 1.00000 \end{pmatrix}$$

5 Mathematica Simulation Results

In order to visualize these results, we created a program using Mathematica/Wolfram Programming Language[3] to simulate particles traveling through this transfer matrix. To do this, we generated 100,000 input vectors (spatial width of 1 mm, angular divergence of 1 mrad, and a momentum byte of 0.5%) and multiplied them by the transfer matrices. From here, we were able to create plots analyzing the spatial distributions of the beam, and of greater importance, analyze the correlation between x and dp/p.

LAPPD 1 location (z=110.708 m)

The following results are at the upstream LAPPD location.

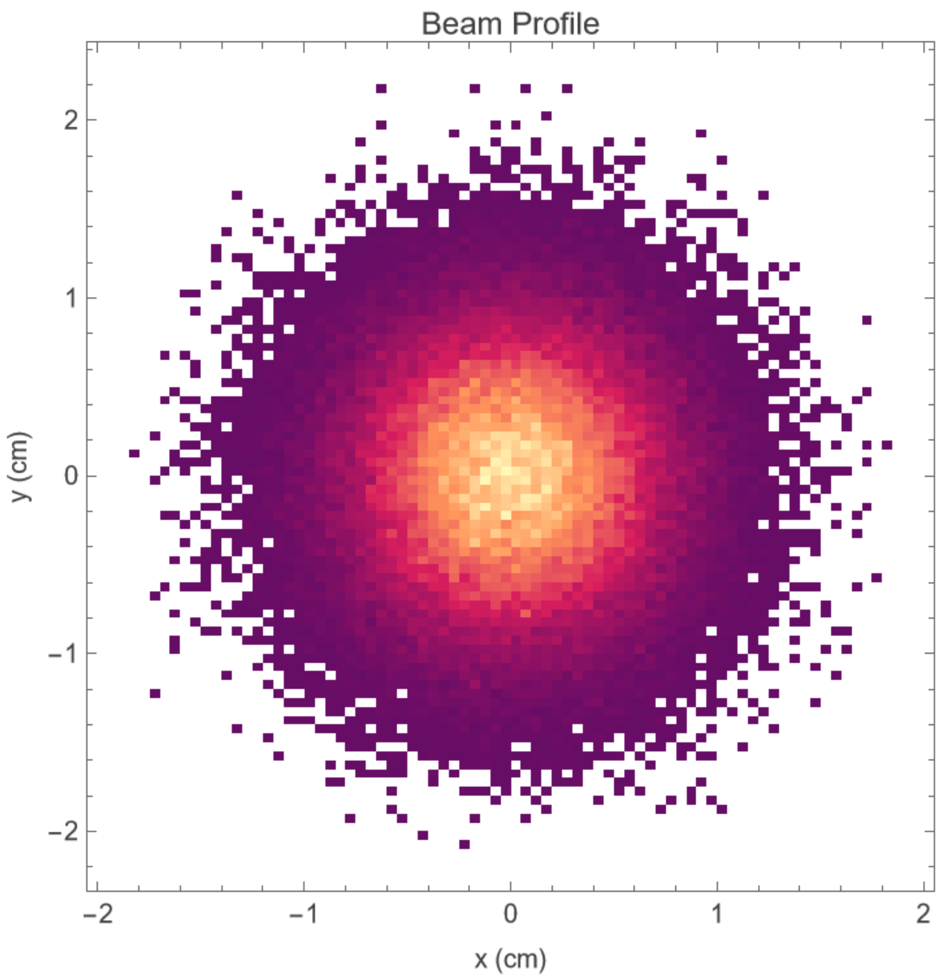


Figure 5: Spatial distribution of the beam at LAPPD 1 location

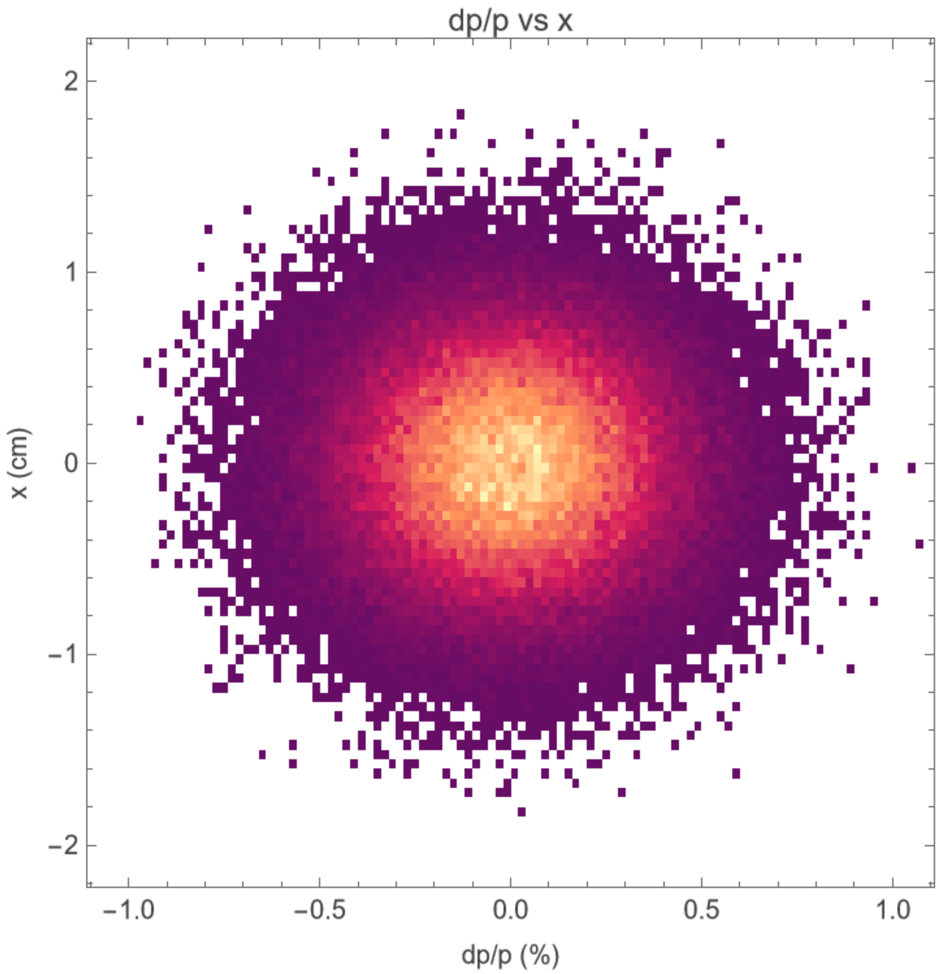


Figure 6: dp/p (%) vs x (cm) at LAPPD 1 location

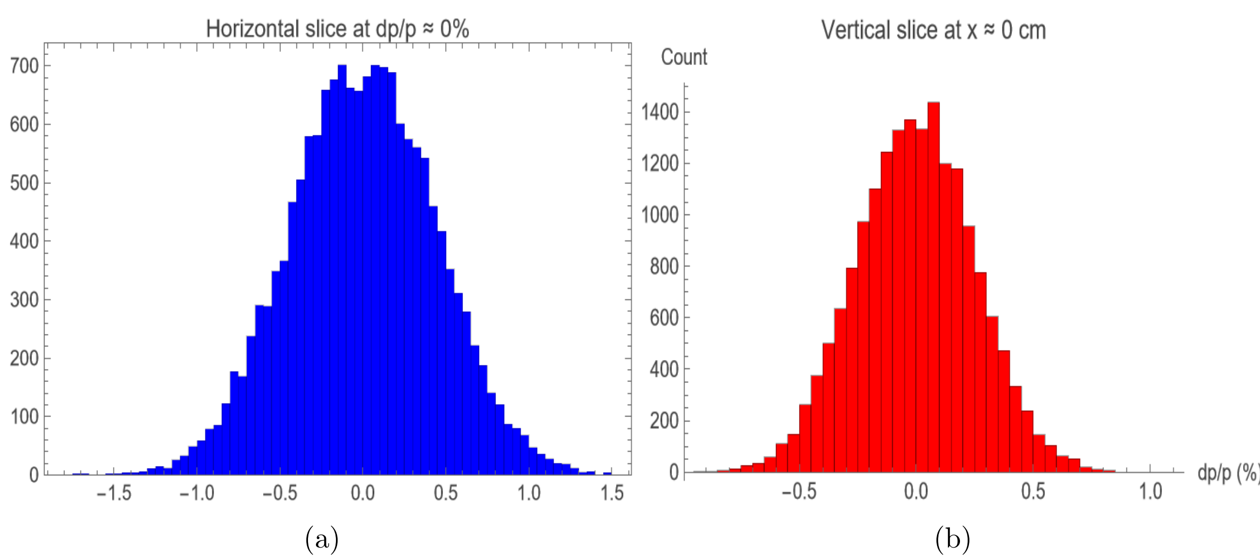


Figure 7: 1-dimensional "slices" of the origin of the dp/p vs x plot. Distribution of dp/p at x=0 (left) and distribution of x at dp/p=0 (right)

From these, we can see the effect of the dispersive element of the transfer matrix, which is -0.00022. While not nonexistent, it is indeed very slight. We can calculate the full dispersive spread by multiplying the (1,6) element by the final momentum byte (calculated using the FWHM of the figure below).

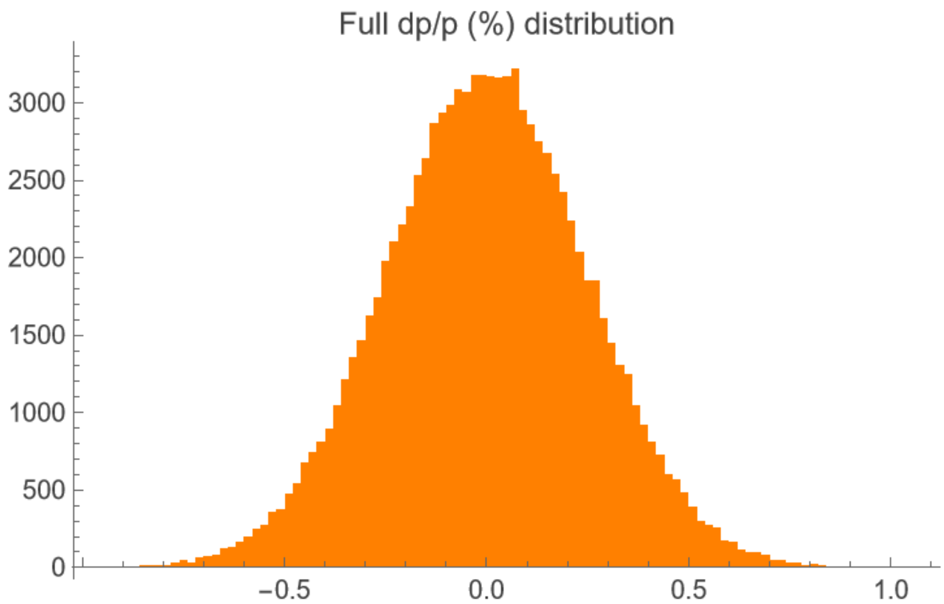


Figure 8: Distribution of dp/p (%) at LAPPD 1 location

$$(-0.00022cm/\%) * (\pm 0.6\%) = \mp 0.000132cm \tag{1}$$

This dispersive effect is very small compared to the actual spatial width of the beam, hence why the diagonal correlation between dp/p and x is not extremely visually evident.

LAPPD 2 location (z=134.147)

The following results are at the downstream LAPPD location.

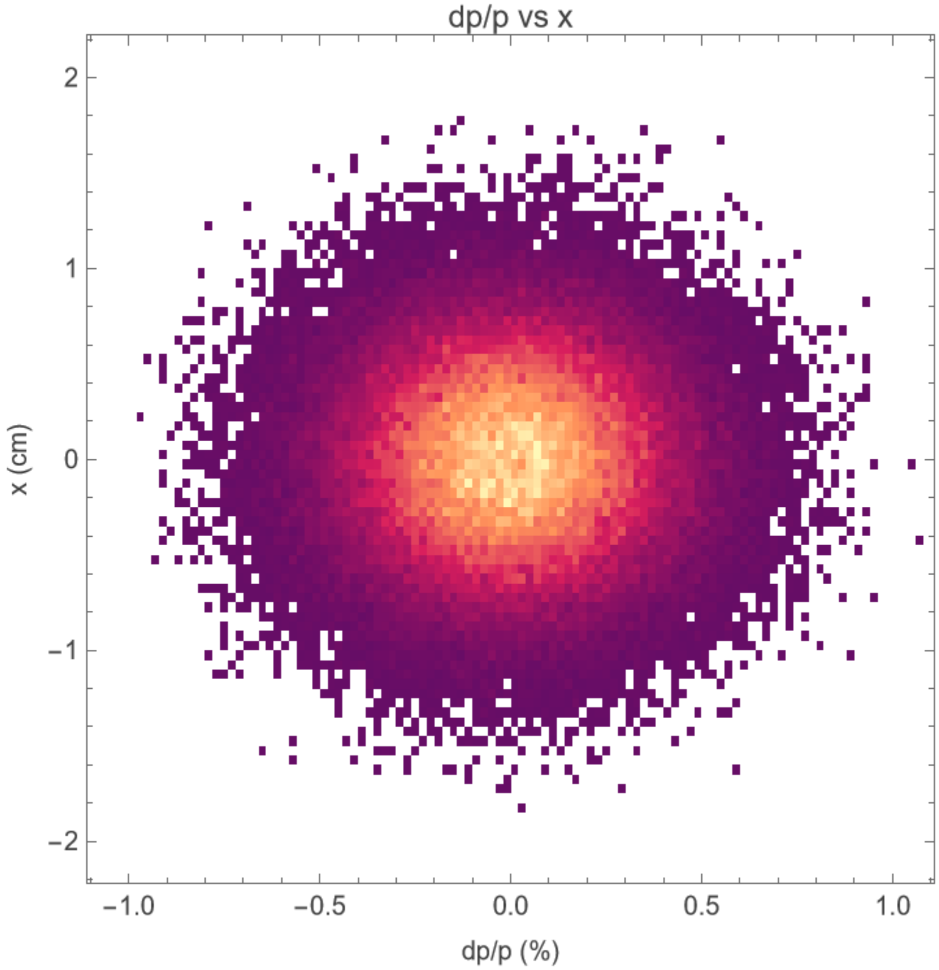
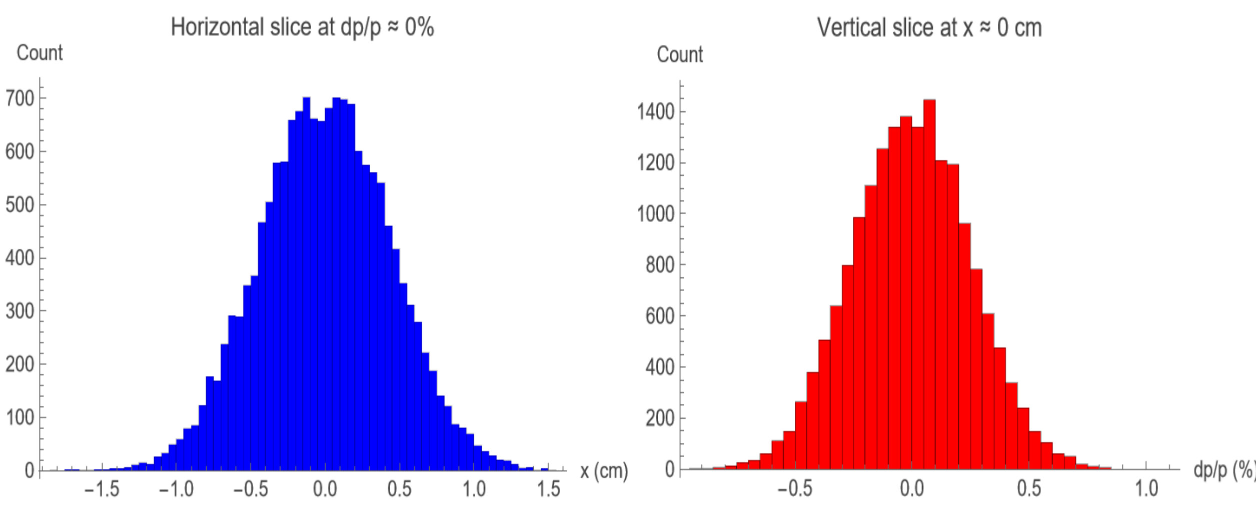


Figure 9: dp/p (%) vs x (cm) at LAPPD 2 location



(a)

(b)

Figure 10: 1-dimensional "slices" of the origin of the dp/p vs x plot. Distribution of dp/p at x=0 (left) and distribution of x at dp/p=0 (right)

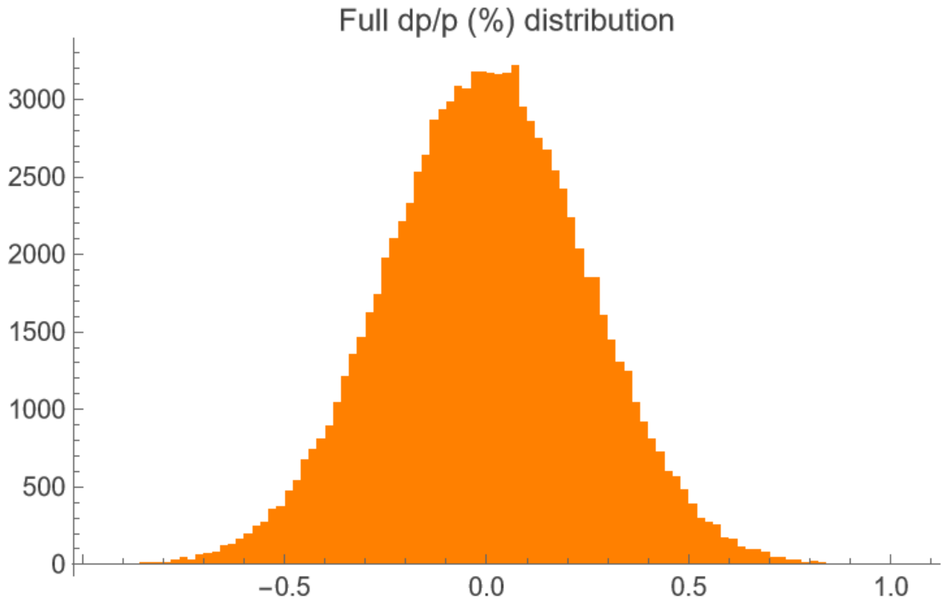


Figure 11: Distribution of dp/p (%) at LAPPD 2 location

We can calculate the dispersive spread at this location as well:

$$(-0.00090cm/\%) * (\pm 0.6\%) = \mp 0.0054cm \tag{2}$$

6 Appendix

Transport Input Code

```
/*PLOT* Test/
 0
 1. 0.01 0.0001 0.01 0.0001 0.0 0.25 1000 /BEAM/ ;
 3. 2.269 /D/ ;
 5. 3.048 1.9415 3.65 /4Q1/ ;
 3. 0.336 /D/ ;
 4. 3.048 .083125 0.0 /4W1/ ;
 3. 10.501 /D/ ;
 5. 3.048 -1.131625 3.65 /4Q2/ ;
 3. 0.635 /D/ ;
 4. 3.048 .090625 0.0 /4W2/ ;
 3. 0.661 /D/ ;
 5. 3.048 -1.131625 3.65 /4Q3/ ;
 3. 2.597 /D/ ;
 3. 4.401 /D/ ;
 3. 9.832 /D/ ;
 5. 3.048 0.38075 3.65 /4Q4/ ;
 3. 0.375 /D/ ;
 3. 1.268 /D/ ;
 5. 3.048 -0.56825 3.65 /4Q5/ ;
```

```
3. 0.341 /D/ ;
3. 1.647 /D/ ;
5. 3.048 .607075 3.65 /4Q6/ ;
3. 3.84 /D/ ;
4. 3.048 -.09828125 0.0 /5e1/ ;
3. 0.491 /D/ ;
4. 3.048 -.09828125 0.0 /5E2/ ;
3. 0.467 /D/ ;
4. 3.048 -.09828125 0.0 /5E3/ ;
3. 0.496 /D/ ;
4. 3.048 -.09828125 0.0 /5E4/ ;
3. 0.483 /D/ ;
4. 3.048 -.09828125 0.0 /5E5/ ;
3. 18.387 /D/ ;
5. 3.048 .159592735 3.65 /5Q1/ ;
3. 0.597 /D/ ;
5. 3.048 -.319185469 3.65 /5Q2/ ;
3. 5.364 /one/ ;
3. 23.439 /two/ ;
SENTINEL
SENTINEL
```

The first line of input (here, a 0) signifies that this is a Transport input file. The following line specifies the initial phase-space coordinates of the beam, as well as the reference momentum in MeV. Each subsequent line represents an individual element or drift in the beamline. The last two drifts go up to the locations of the LAPPDs. For more information on Transport input codes, see the manual[6].

7 Acknowledgements

Thank you to Joe Pastika and Sue McGimpsey from FTBF as well as Evan Angelico for helping me throughout this project with various challenges in coding and documentation.

References

[1] E. Angelico, *Development of Large-Area MCP-PMT Photo-Detectors for a Precision Time-of-Flight System at the Fermilab Test Beam Facility*, Ph.D. dissertation, Department of Physics, The University of Chicago, Chicago, Illinois, 2020.

[2] U. Rohrer, *PSI Graphic Transport Framework*, based on a CERN-SLAC-FERMILAB version by K.L. Brown et al, Paul Scherrer Institut (PSI), Villigen, Switzerland.

[3] Wolfram Research, Inc., Mathematica, Version 14.3, Champaign, IL (2025).

[4] Rinn, Tim, *A G4BEAMLINE simulation of the Meson Test Beam*, Fermilab Test Beam Facility. https://web.fnal.gov/experiment/FTBF/Simulations%20Documents/G4BeamlineStudy.pdf

[5] https://ftbf.fnal.gov/mtest-beam-areas/

[6] https://lss.fnal.gov/archive/nal/fermilab-nal-091.pdf

[7] https://psec.uchicago.edu/library/applications/fermilab_test_beam/Proj%207002%20Req%207070%20As-found%20Survey%20of%20MINERvA%20Test%20Beam.pdf

[8] https://docs.google.com/spreadsheets/d/1-Fokpj0ncVAJP4tRIdsKxrOHDZSHIjUy/edit?gid=1440020616#gid=1440020616